

# EDQNM closure: "A homogeneous simulation to support it", "A quasi-homogeneous simulation to disprove it"

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It is known that two-point closures are useful tools for understanding and predicting turbulence. Among the various closures, the Eddy Damped Quasi-Normal Markovian (EDQNM) approach is one of the simplest and, at the same time, most useful. Nowadays, direct numerical simulations (DNS) can provide information that can be used to test the validity of two-point theories. It is the purpose of the present work to use DNS to validate, or improve upon, EDQNM.

In the first part of this work ("a homogeneous simulation to support it") we selected a case for which EDQNM is known to give satisfactory results: homogeneous isotropic turbulence. We then evaluated quantities which may be used to test the assumptions of two-point closure approximations: spectral Lagrangian time scales. Our goal is to make a careful and refined study to validate (and possibly improve) the EDQNM theory.

The aim of the second part of the work ("a quasi homogeneous simulation to disprove it") is, on the contrary, a test of EDQNM in a much more difficult situation. Our purpose is to build a reference case for which EDQNM is likely to give poor results. We present an attempt to generate a quasi-homogeneous turbulent field containing "organized" structures, by artificially injecting them in the initial conditions. The results of direct simulations using such initial conditions are expected to provide a challenge for EDQNM since this kind of field is simple enough to allow comparisons with two-point theories, but at the same time contains "coherent" structures which cannot be expected to be accurately accounted for by closures based on expansions about Gaussianity.

## 1. Lagrangian spectral times in isotropic turbulence

### 1.1 The Assumptions of EDQNM

In the EDQNM theory the growth of the third order moments is limited by the introduction of a damping term in the rate equation for the triple correlations. The damping coefficient  $\mu(k, p, q)$  is essentially an inverse time scale for the decay of the triple correlation among the three wave-vectors of a triad. It is generally assumed that  $\mu(k, p, q)$  is the sum of the inverse time scales for the individual wavenumbers:

$$\mu(k, p, q) + \nu(k^2 + p^2 + q^2) = \eta(k) + \eta(p) + \eta(q)$$

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and  $\eta(k)$  is specified phenomenologically.

For flows at Reynolds numbers sufficiently high to contain an inertial subrange, the damping coefficient must have the form (Orszag 1970)

$$\eta(k) = a\epsilon^{\frac{1}{2}} k^{\frac{2}{3}}$$

The form proposed by Pouquet et al (1975) for an arbitrary energy spectrum:

$$\eta(k) = \lambda \left[ \int_0^k p^2 E(p) dp \right]^{\frac{1}{2}} + \nu k^2$$

is more commonly used. The value of constant  $\lambda$  is generally taken to be 0.355.

According to this expression the time scale at wavenumber  $k$  depends only on wavenumbers smaller than  $k$  (i.e. only the large eddies influence the damping). There is however no reason, on physical grounds, to assume that the effect of the smaller scales can be neglected. Indeed, in order to devise a two-point closure compatible with the RNG approach of Yakhot and Orszag, Kraichnan (1987) recently introduced a model in which it is assumed that only the small scales affect the damping. In fact, neither type of expression can be completely correct, and during his stay at the Center for Turbulence Research (CTR) 1987 summer program Kraichnan suggested the following possibility of building a time scale that depends on all scales:

$$\eta(k) = \lambda_a \left[ \int_0^k p^2 E(p) dp \right]^{\frac{1}{2}} + (\nu + \nu_>(k)) k^2$$

where  $\nu_>(k)$  is given by :

$$\nu_>(k) = \lambda_b \int_k^\infty \frac{E(p) dp}{\eta(k) + \eta(p)}$$

where  $\lambda_a$  and  $\lambda_b$  are constants. It can be shown that these two new constants must satisfy the following relation:

$$\lambda_a = \lambda - 2(1 - \log 2) \frac{\lambda_b}{\lambda}$$

in order that the new form reduces to the old one in the case of an inertial range.

During the CTR Summer Program, we tested the new form for the damping by introducing it into the EDQNM computation code written by Orlandi. We found no significant differences from the results obtained using the classical form, as long as  $\lambda_a/\lambda$  remains larger than 0.5. We noticed a small effect on the skewness, (see figure 1); the other quantities remain unaffected.

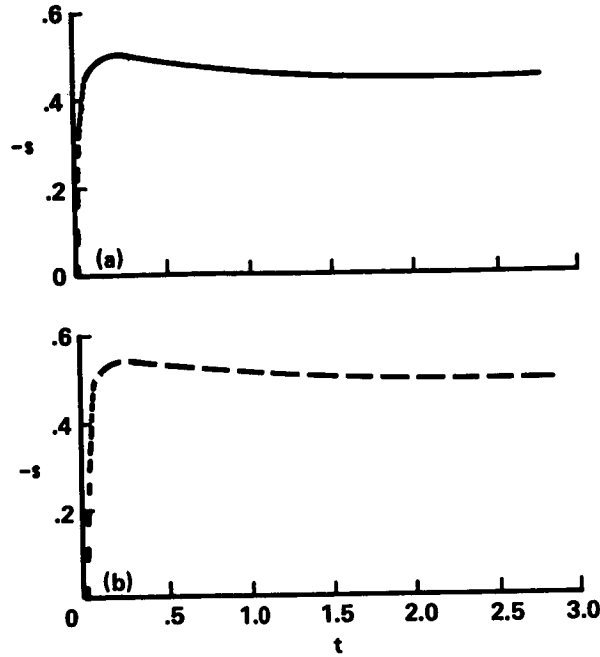


FIGURE 1. Evolution of the skewness in time. Comparison of EDQNM results obtained with different formulations for the damping. (a) damping given by Pouquet et al (1975); (b) damping suggested by Kraichnan (with  $\lambda_a/\lambda = .5$ ).

### 1.2 Evaluation of $\eta(k)$ using direct numerical simulation.

In order to test the expression for  $\eta(k)$  used in EDQNM against actual turbulence time scales, we have to deduce a spectral time scale from the simulation results. Comparisons between EDQNM and the Direct Interaction Approximation suggest evaluating the time scale using two-time correlations of the velocity field. The proper time scale must be invariant under an arbitrary Galilean transformation since the triple correlations must not be affected by convective effects. The time scale must therefore be a Lagrangian time scale derived from Lagrangian two-time correlations, as suggested by LHDIA. For practical reasons, we used the two-time correlations introduced by Kaneda (1981):

$$R_{ij}(\mathbf{x}, t' | t; \mathbf{x}', t' | t') = \langle u_i(\mathbf{x}, t' | t) u_j(\mathbf{x}', t' | t') \rangle$$

where  $u_i(\mathbf{x}, t' | t)$  is the velocity at time  $t$  of the fluid particle whose trajectory passes through  $\mathbf{x}$  at time  $t'$ . One could also use the two-time correlations defined by Kraichnan in the LHDIA or ALHDIA theories, or work with the response tensor, but the evaluation of Kaneda's correlations is simpler.

Spectral information is obtained by Fourier transforming with respect to the initial position of the particles.

To deduce the Lagrangian correlations from the results of a DNS, one has to

follow particle trajectories. This is done using Squires' code, which was developed for the study of particle dispersion.

### *1.3 Results*

Two direct numerical simulations of homogeneous isotropic turbulence were used. Both used Rogallo's code. In the first case, a  $64 \times 64 \times 64$  grid was used. In the second, a  $128 \times 128 \times 128$  grid was introduced in order to increase the Reynolds number.

We present here only a limited number of results. For a more detailed study see the paper by Lee, Squires, Bertoglio, and Ferziger (1987). A study preliminary to this work was performed in Lyon using Large Eddy Simulation on a  $16 \times 16 \times 16$  grid.

In the DNS, it would have been desirable to place particles at each point of the computational grid, but doing so would have increased the cost of the computation prohibitively. The number of markers is therefore much smaller than the number of grid points. Marker placements were of two types. First, to ensure accurate large scale statistics, markers were placed on a  $16 \times 16 \times 16$  equally spaced grid. This provides a set of widely spaced particles. To obtain data on the small scales, a second set, consisting of  $8 \times 8$  lines of 64 particles was used.

In figure 2a, the Lagrangian correlations obtained from the  $128 \times 128 \times 128$  run are given as functions of the dimensional separation time  $t - t'$ . As expected, the correlations corresponding to large wavenumbers decrease rapidly.

The correlations can also be plotted as functions of the time normalized by the candidate time scale; an accurate time scale should collapse the results to a single curve. figure 2b shows that, when the classical time scale is used, the collapse is good at high wavenumbers but not as good at low wavenumbers. When Kraichnan's expression is used (with  $\lambda_a = \lambda/2$ ), the collapse is slightly improved at low wavenumbers, and remains acceptable at high wavenumbers (figure 2c).

### *1.4 Problems and future orientations*

The major problem encountered is the lack of sufficient sample at high wavenumbers. The number of particles used ( $8 \times 8 \times 64$ ) was probably not large enough to allow conclusive results concerning the highest wavenumbers in the simulation.

In the comparison between Lagrangian and Eulerian correlations, another problem appears. As expected, for short times and at high  $k$ , the Eulerian correlation decreases faster than the Lagrangian one; however, for long times, the trend appears to be reversed. In fact, for large values of the separation time, the significance of Fourier transforming with respect to the initial positions of the markers must be questioned. One could try to use another definition for the spectral time scale in this case. However, we believe that information deduced from the small time behavior is sufficient for the purposes of our study.

It would be interesting to extend the present study to anisotropic flows, as almost no information is available about the effects of mean velocity gradients or anisotropy on the damping. The easiest anisotropic case to investigate would be turbulence submitted to uniform solid body rotation since, in this case, steady coordinates (in the rotating frame) can be used. Due to the lack of time, we could not investigate this case during the summer program. It should be the subject of future work.

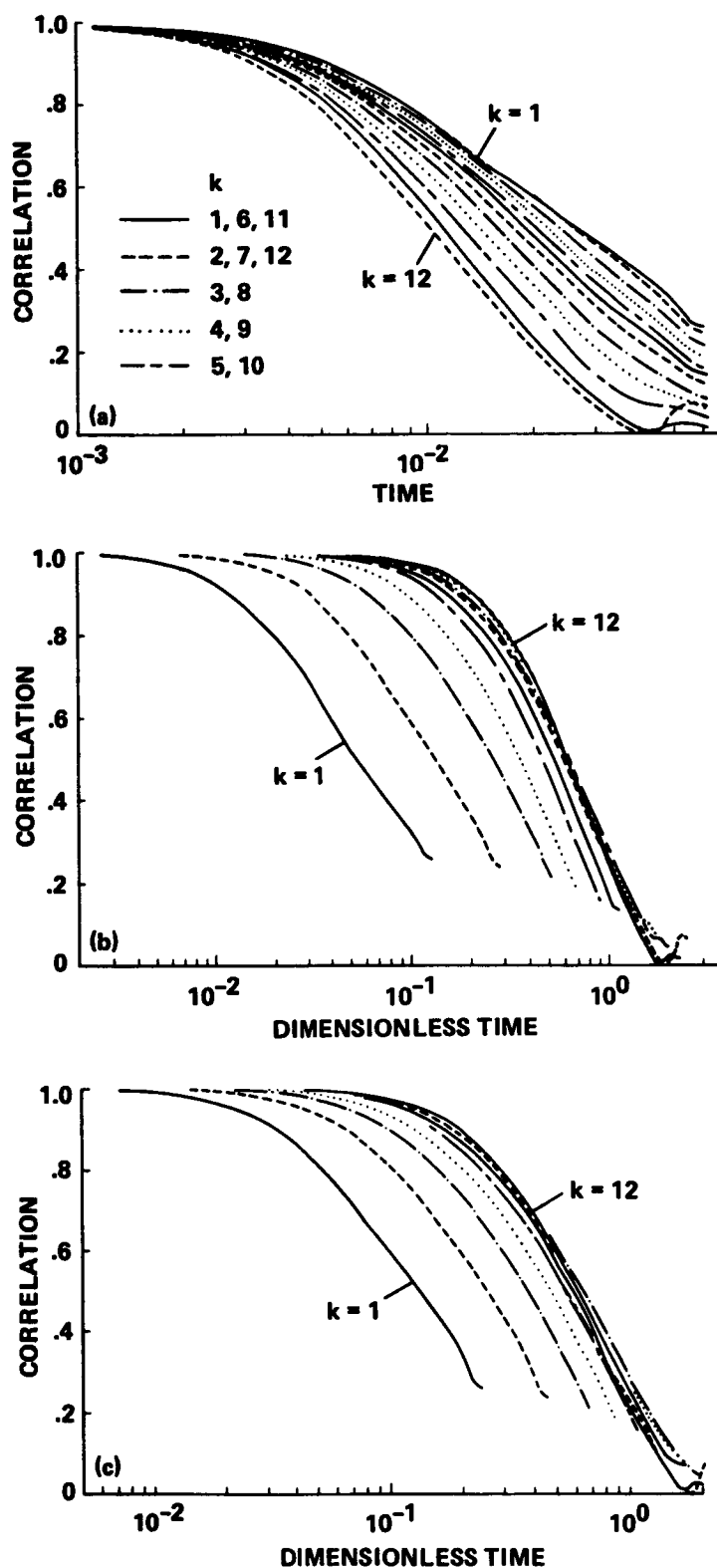


FIGURE 2. Two-time Lagrangian correlations as a function of the time separation at different values of the wavenumber. (a) un-normalized time separation, (b) time separation normalized using Pouquet's formula, (c) time separation normalized using Kraichnan's formula.

In the case of isotropic fields, interest in studying the damping term is quite limited, since it is known that the existing formulations lead to satisfactory predictions of the decay of isotropic turbulence. This is, to a certain extent, true, (and is the reason why our goal is the study of anisotropic turbulence). However we believe that, even though limited to isotropic turbulence, the results of the present study are valuable. In the study of homogeneous isotropic turbulence containing a gap or a peak in the spectrum, for example, the accuracy of the expression for the damping is of greater importance. Since the choice of the damping term has an important effect on the "localness" of the triadic interaction in  $k$  space, a better expression for  $\eta$  could provide interesting information concerning localized interactions and, possibly, insight about the effect of the presence of coherent structures on the energy cascade (see Kraichnan, 1987). Furthermore, we believe that, in large-eddy simulation, the influence of the subgrid terms on the Lagrangian time scales should be investigated.

## 2. A quasi-homogeneous field containing "organized" structures

### *2.2 Aim of the study and initial conditions.*

The first part of the work was devoted to the study of a case in which EDQNM is known to perform well: isotropic turbulence. The aim of the second part is significantly different. It consists of an attempt to simulate a case which, although simple enough to be handled by closures with reasonable computation times, is likely to provide a severe test of EDQNM.

Our goal is to get insight into the ability of two-point closures to account for situations in which coherent structures are present. How are the predictions of EDQNM affected by the presence of organized eddies?

The existence of coherent structures in wall bounded flows is well known. Wall bounded flows would, however, require a large amount of computational effort to be predicted by EDQNM, and at least at first, simpler situations have to be studied.

The basic idea of the present work is therefore to artificially inject coherent structures into a "quasi-homogeneous" turbulent field (quasi-homogeneous meaning periodic rather than homogeneous).

During the summer program we tried to define initial conditions for a simulation of a flow containing "coherent" structures in a periodic cubic box. The results of such a simulation could provide reference data to test EDQNM.

In an attempt to create such a turbulent field, an experiment has been devised in Lyon. This experiment consists of generating organized structures in grid turbulence by using a grid equipped with small rotating propellers. One hundred counter-rotating propellers are used in this experiment (one at each node of the grid); see figure 3. The results (see Michard et al, 1986) display interesting behavior, in particular, a spike appears in the spectrum; see figure 4.

Large Eddy Simulations on  $16 \times 16 \times 16$  grids, run in Lyon, using initial conditions containing four vortices embedded in a random field, show that some experimental tendencies can be predicted, and, to a certain extent, understood (for example, the existence of a periodic distribution of anisotropy). However, due to the lack

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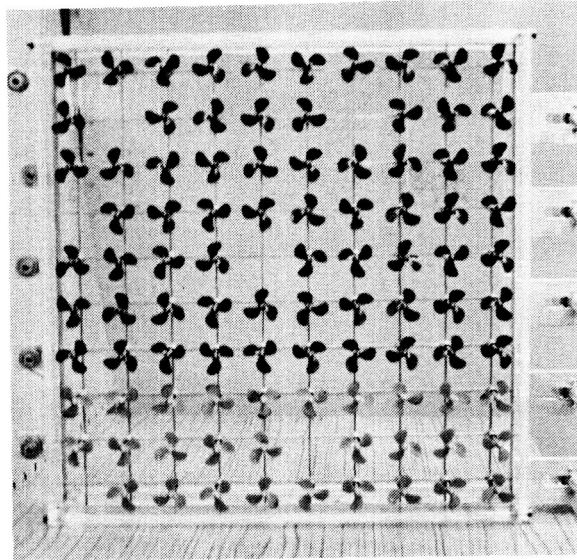


FIGURE 3. Grid equipped with propellers (Michard et al,1986).

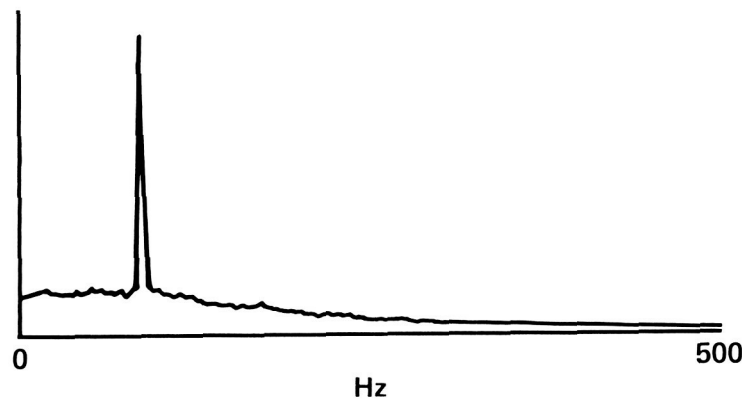


FIGURE 4. Experimental one-dimensional energy spectrum of Michard et al (1986).

of resolution of this coarse grid, it has not been possible to inject a peak in the spectrum in the flow direction.

The work done at CTR consists in building an initial field that contains such a peak. We used a  $64 \times 64 \times 64$  mesh. A few simulation runs were started to test the initial field. After several attempts, it was decided to define the initial conditions in terms of the vorticity field. In the  $64 \times 64 \times 64$  box, four vortex-containing cells, corresponding to the wakes of four propellers, were defined. In each cell three vortices are introduced, centered on helical lines corresponding to the wakes of the three blades of each propeller. In order to ensure zero circulation in each cell, another vortex was introduced on the centerline. A representation of this initial field is given in figure 5.

This field was embedded in a random fluctuating field. For this random field, we

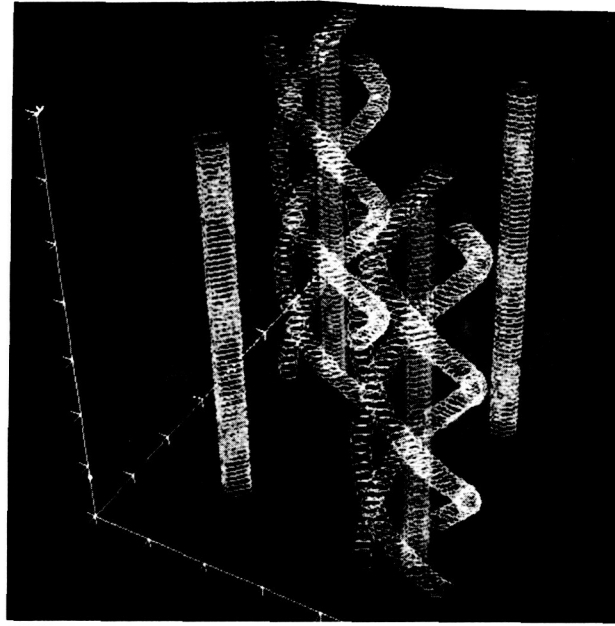


FIGURE 5. Representation of the initial “coherent structures” used in the simulation. These are the idealized wakes of the counter-rotating propellers shown in figure 3.

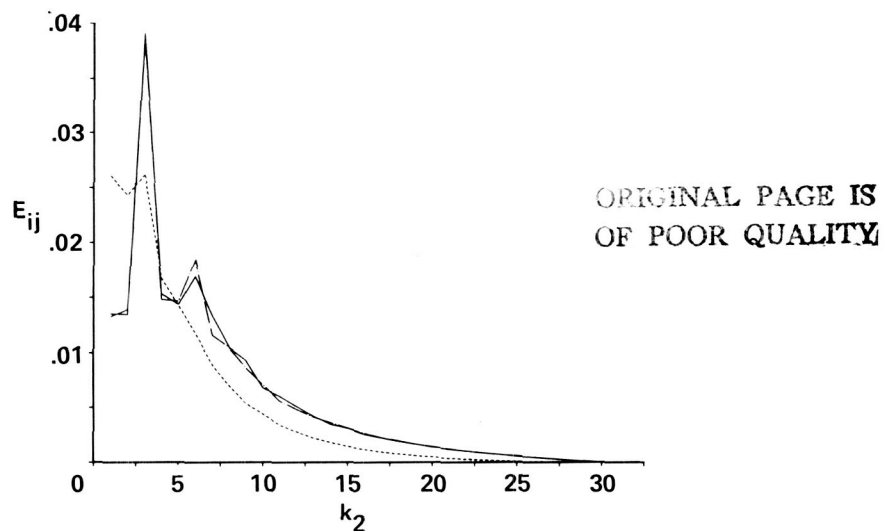


FIGURE 6. Initial one dimensional spectra (vortex field + isotropic turbulence, averaged over the entire  $x_1 - x_3$  space). The mean flow is in the  $x_2$  direction. —  $E_{11}$ , ----  $E_{22}$ , — —  $E_{33}$ .

used the one created by Lee & Reynolds (1985) with  $N = 64$ , and  $Re = 51.13$ .



The initial spectrum, in the  $k_2$  direction, is given in figure 6. A small peak appears at the wavenumber corresponding to the blade passing frequency.

### 2.2 Results and directions for future work.

The peak in figure 6 is relatively weak, although the energy contained in the vorticity field is not small compared to the total energy of the field. However, it must be pointed out that the plotted spectrum is an average over the X,Z plane, and one should investigate the behavior of the one-dimensional spectra obtained at a given point in the X,Z plane.

It is worth noting that the peak appears at the blade passing frequency (due to the construction of the initial conditions) whereas the experimental spike is found at the rotation frequency of the propellers. During the simulation, the computed peak evolved only slightly and no subharmonic spike appeared at the rotation frequency. The simulations were, however, limited to one hundred time steps due to time constraints; longer runs are necessary to see the expected behavior.

Another experimental feature is that there is an axisymmetric contraction downstream of the grid. This axisymmetric contraction was introduced to amplify the intensity of the peak. Without it, the peak would die immediately; in fact, the amplification was much stronger than expected. The axisymmetric contraction should probably be included in the simulation. It is hoped that this work will be completed in the near future. It could provide a challenge for EDQNM, as preliminary studies show that the experimental decay of the peak is overpredicted by the closure. The existence of a reference field deduced from the simulation would be highly valuable since precise analysis of the mechanism occurring in the experiment is hard to deduce from the measured quantities.

### 2.3 Acknowledgements

The authors would like to thank all the local participants to the Summer Program (at NASA Ames, at CTR and at Stanford University) for their considerable help during the summer and the large amount of work they did for us. In particular, we want to thank P. Moin, W. C. Reynolds, and J. Kim for the perfect organization of this meeting. The authors would also like to thank P. Orlandi, R. Rogallo, and P. Spalart for helpful assistance.

Finally, we would like to thank J. C. R. Hunt, R. Kraichnan and C. Leith for illuminating discussions during the course of this work.

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